Prepared by:

**Stuart Silverman**
FSA, MAAA, CERA

**Philip Simpson**
FIA, ASA, FSAI

October 2011

Case Study:

# Modelling Longevity Risk for Solvency II

Milliman

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## ACKNOWLEDGEMENTS

The following Milliman consultants made a significant contribution to the development of this case study:
Robert Bugg, Ghali Elboukfaoui, Farzana Ismail, and Daniel Theodore.

## INTRODUCTION

Many companies have expended significant time and effort as they prepare to comply with the forthcoming major change in the EU insurance regulatory regime to Solvency II. Many of these companies have focused on the development of *internal models* by implementing stochastic projections focusing primarily on the market risk (i.e., asset volatility) to capture much of the volatility associated with assets (including asset-related liability risk). In its report on the most recent Quantitative Impact Study 5 (QIS5), the European Insurance and Occupational Pensions Authority (EIOPA) reported, "Life underwriting risk is the second most material module for life undertakings behind market risk. Within this lapse risk and longevity risk are the two most material submodules."[1] Given this assessment, many companies are now evaluating the use of internal models for longevity risk.

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This case study illustrates the potential benefit to annuity writers of reflecting mortality rate volatility in the liability calculations of internal models. We describe a case study that demonstrates the calculation of required capital under our interpretation of Solvency II[2] utilizing an internal model that reflects volatile future mortality rates and we examine the required capital calculation for a block of in-payment annuities.

Since the intent of this case study is to focus on liabilities and highlight the value of modelling mortality rate volatility by developing required capital specifically associated with longevity risk, we intentionally exclude any recognition of market risk.

The results of this case study support our assertion that internal models utilizing stochastic liability projections with volatile mortality assumptions may be valuable to companies trying to understand and manage their capital requirements.

To generate the values in this analysis, we used Milliman's longevity risk projection system, REVEAL.[3] REVEAL is a proprietary software platform available to Milliman clients that performs stochastic projections utilizing volatile mortality assumptions:

- We utilize REVEAL's functionality to compare capital requirements of the Solvency II Standard Formula (Standard Formula Capital Requirement) to those calculated under a principles-based capital calculation (Economic Capital Requirement).
- We use REVEAL to evaluate the Solvency II capital requirement when using a sophisticated internal model (Internal Model Capital Requirement) and compare that to the Standard Formula Capital Requirement.
- We illustrate these various capital requirements under various assumption sets.

A major observation in this case study is that the use of a stochastic-based internal model reflecting volatility in future mortality rates may allow insurance companies to hold less required capital under Solvency II on their longevity-risk-related products than required by the Solvency II Standard Formula.

1 EIOPA (2011). Report on the fifth Quantitative Impact Study (QIS5) for Solvency II, p.77.

2 This paper focuses on longevity issues. A debate remains on the treatment of illiquidity risk. Therefore we have adopted our interpretation of the QIS5 methodology.

3 Risk and Economic Volatility Evaluation of Annuitant Longevity, or REVEAL, is a system developed to analyze longevity risk. REVEAL generates stochastic projections on pension and annuity liabilities with volatile assumptions (i.e., baseline mortality, mortality improvement, extreme mortality and longevity events, and plan participant behavior–such as retirement dates and benefit elections). For more information about REVEAL, please see http://www.milliman.com/expertise/life-financial/products-tools/reveal/.

## BACKGROUND

Beginning in 2014,[4] and pending approval by the European Parliament, insurance companies in Europe will have to satisfy the three *Pillars* of Solvency II, covering (1) capital requirements, (2) corporate governance, and (3) reporting and disclosure. Solvency II is designed to provide common financial regulation across the EU, improving consumer protections and facilitating cross-market competition.

The first pillar of Solvency II involves two levels at which capital is measured: The Minimum Capital Requirement (MCR) and the Solvency Capital Requirement (SCR). Although the detailed specification of its calculation is not yet finalized, the MCR is the floor at which a company is assumed to be at serious risk of default and will be subject to immediate regulatory action.

In the less extreme case, the SCR is used to determine if a company has sufficient cushion to handle a reasonably high level of unexpected losses (i.e., based on a confidence level of 99.5%) over a one-year time horizon, called a 1-in-200-year event. The SCR is designed to reflect market risk, credit risk, non-life underwriting risk, life underwriting risk, and operational risk. The company may calculate the SCR by either *Standard Formulas* or with a regulator-approved internal model. In exchange for simplicity, the Standard Formulas may implicitly contain substantial margins (possibly in excess of those utilized by internal models), which can result in higher capital requirements (compared to those calculated using the internal models).

Within the longevity risk sub-module, the total asset required under Solvency II is the sum of the best estimate liability, the SCR, and a risk margin. While the risk margin is part of the technical provisions, another way of considering this formula is that Solvency II requires capital held in excess of the best estimate liability equal to the SCR and a risk margin.

Many European insurance companies have significant longevity risk exposure (in the form of in-payment annuities). This includes a significant number of pension schemes that, having been closed to future accruals, were *bought out*, moving all ongoing liabilities off the employers' books. Although Solvency II requirements do not cover pension liabilities held by non-insurance companies, Solvency II may form a template for possible future pan-European pension solvency regulation. Hence, Solvency II will require many European companies to address new capital requirements, specifically with regard to longevity risk. As a result, these insurers are working to develop financial models and assumptions to meet Solvency II requirements.

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Under the Solvency II requirements, an insurer may substitute the results from a *regulator-approved internal model* to calculate the SCR in place of the Standard Formula. The internal model must satisfy certain general standards, the details of which are still being explored, including:

- The **use test** expects that the internal model is *widely used and plays an important role in*[5] the organization for the purposes of analysis and decision-making. The underlying data and output should be relevant for and familiar to management.[6]

- The model should be sufficiently sophisticated and fully developed to support the standards of **statistical quality**.[7]

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4 This is the expected date from the Omnibus II directive, but there is a current debate on some form of phased introduction over 2013.

5 Financial Services Authority (September 2008). *Insurance Risk Management: The Path to Solvency II.*

6 Solvency Capital Requirement – Full and Partial Internal Models, Article 118, Subsection 3.

7 Ibid., Article 119

- The model must satisfy **calibration standards** to relevant current internal and external statistics so that it adequately captures recognizable trends and volatility.[8]

- The internal model should be able to be back-tested to demonstrate how **profit and loss attribution** can be traced back to its sources.[9]

- To meet **validation standards**, regular checks should be run to provide an ongoing demonstration of the quality of its estimates.[10]

- The system must have sufficient **documentation**, including detailed descriptions of the limits and deficiencies of the model.

The internal model standards have been discussed in more detail elsewhere and an in-depth discussion about internal models is outside the scope of this paper.

8 Ibid., Article 120
9 Ibid., Article 121
10 Ibid., Article 122

# DESCRIPTION OF THE HYPOTHETICAL PORTFOLIO AND BEST ESTIMATE ASSUMPTIONS

In this section of the paper, we describe the hypothetical in-payment annuity portfolio and the best estimate assumptions that are used in the case study.

1. **The Valuation Date** (and measurement date for the portfolio) is December 31, 2010.

2. **Population**

   The hypothetical portfolio consists of 50,000 in-payment annuities (Hypothetical Portfolio). The Hypothetical Portfolio was designed to reflect a variety of different characteristics to be consistent with a typical block of in-payment annuities held by an insurer. Further, the Hypothetical Portfolio is sufficiently large to minimize random *basis* volatility, thus allowing the analysis to highlight the effect of other volatility factors.

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Several characteristics of the Hypothetical Portfolio are shown in Figure 1.

**FIGURE 1: DISTRIBUTION OF HYPOTHETICAL PORTFOLIO AT VALUATION DATE**

**(BASED ON CURRENT ANNUALIZED BENEFIT PAYMENTS)**

| MEASURING LIFE | |
|---|---|
| PRIMARY ANNUITANT | 67% |
| SPOUSE (WIDOW/WIDOWER) | 33% |
| **ANNUAL BENEFITS** | |
| INDEXED TO CPI | 84% |
| FIXED | 16% |
| **GENDER OF MEASURING LIFE** | |
| MALE | 55% |
| FEMALE | 45% |

**ANNUALIZED BENEFIT AMOUNT**

| ANNUALIZED AMOUNT | PCT |
|---|---|
| <1K | 23% |
| 1K-5K | 45% |
| 5K-10K | 22% |
| 10K-20K | 7% |
| 20K-30K | 1% |
| 30K+ | 2% |

**JOINT LIFE (OR SURVIVORSHIP) BENEFITS *AS A PERCENTAGE OF ALL BENEFITS IN EACH AGE GROUP***

| PRIMARY LIFE AGE GROUP | PCT |
|---|---|
| 60-64 | 92% |
| 65-69 | 89% |
| 70-74 | 84% |
| 75-79 | 79% |
| 80-84 | 74% |
| 85-89 | 61% |
| 90-94 | 47% |
| 95-99 | 29% |

**AGE OF MEASURING LIFE**

| PRIMARY LIFE AGE GROUP | PCT |
|---|---|
| 60-64 | 13% |
| 65-69 | 22% |
| 70-74 | 21% |
| 75-79 | 19% |
| 80-84 | 14% |
| 85-89 | 7% |
| 90-94 | 2% |
| 95-99 | 1% |

Other assumptions:

- For married beneficiaries, it is assumed husbands are three years older than wives.
- Survivor benefit (to spouse) equals 50% of benefit before death.
- Benefits include a five-year period certain from annuitization, assumed to have occurred approximately when the primary life attained age 60 but no later than April 2010.
- Index-based increases are credited a fixed 5% increase annually in April as a proxy for future inflation.

Additional details about the Hypothetical Portfolio are located in Appendix A.

3. **Best estimate mortality**

   Expected mortality (before improvement) is assumed to be equal to 90% of the PCMA00 and PCFA00 mortality tables, respectively for male and female lives. In the CMI library of mortality projections, the PCMA00 and PCFA00 tables are described as *Life Office Pensioners, Combined, amounts - ultimate*. These are standard UK mortality tables.

4. **Best estimate mortality improvement**

   Male: CMI 2010 projection model, with a long-term rate of 1.2% p.a., applied from 2000 onwards.

   Female: CMI 2010 projection model, with a long-term rate of 0.9% p.a., applied from 2000 onwards.

5. **Discount interest**

   Scenario cash flows are discounted using the risk-free spot rates[11] as of December 31, 2009.

   — For evaluation of the Best Estimate Liability and Solvency Capital Requirement, the risk-free rates contain an allowance for 100% illiquidity premium.

   — When calculating risk margins, the risk-free rates contain no illiquidity allowance.

   The spot rates used are shown in the Appendix B.

6. **Projection and payment mode**

   The values generated in the case study reflect annual reporting of annuity payments and deaths (with the first payment on January 1, 2011).

[11] QIS5 (GBP only).

# DISCUSSION OF STOCHASTIC PROJECTION METHODOLOGY AND VOLATILITY PARAMETERS

In this section of the paper, we briefly discuss the stochastic projection methodology and volatility parameters used to determine the Economic Capital Requirements and Internal Model Capital Requirements used in this study. See Appendix C for a detailed description of the stochastic projection methodology and volatility parameters. Each actuary should use his or her own judgment when developing stochastic projection methodologies, assumptions, and volatility parameters.

The stochastic projections reflect three sources of volatility:

1. Randomized dates of death
2. Future mortality improvement trends volatility
3. Potential extreme longevity occurrences (in excess of historical mortality improvement trend volatility)

### 1. Randomized dates of death

The mortality curve derived of each life in the Hypothetical Portfolio for a particular scenario is derived from the baseline expected mortality assumption, adjusted for any applicable volatility in future mortality trends and extreme longevity occurrences. The date of death for each life is determined using a Monte Carlo simulation applied to the stochastically determined mortality curves.

Specifically, for each life in any given scenario, a uniform random number between 0 and 1 is compared to the cumulative probability of survival for that life: If the random number is less than or equal to the cumulative probability of survival to a given duration, the life remains in the population to the next duration. At the earliest duration that the random number is greater than the cumulative survivorship probability, the model treats that life as having died in that duration (and no further testing is performed on that life).

### 2. Future mortality improvement trends volatility

Historical levels of mortality improvement have not emerged in smooth and predictable trends. Mortality improvement may be perceived as a combination of long-term waves with lingering effects over multiple years, and apparently more random annual fluctuations. The pattern of mortality improvement is important because, when determining capital requirements, we are less concerned with modelling mortality improvement statistics, and are more concerned with resulting year-to-year cash flows. While we acknowledge the work that has been done developing assumptions for expected mortality improvement, an inspection of historical experience reveals discontinuities and irregular fluctuations. The present value of projected benefit payments is affected by this volatility in the rates of mortality improvement.

**The pattern of mortality improvement is important because, when determining capital requirements, we are less concerned with modelling mortality improvement statistics, and are more concerned with resulting year-to-year cash flows.**

To project future mortality improvement volatility, we utilize historical levels of general UK population data over the period of years from 1979 to 2009, specifically focused on three factors:

1. **Long-term mortality improvement trend volatility**: We observe in historical data that mortality improvement exhibits measurable long-term trends. The long-term movements may be the result of various factors, including events in medical practice, medical research, economic shifts, political activities, and environmental changes.

   For this case study, our projected long-term mortality improvement volatility was assumed to cover 10-year periods, based on volatility parameters determined from historical levels of mortality improvement volatility over consecutive 10-year intervals.

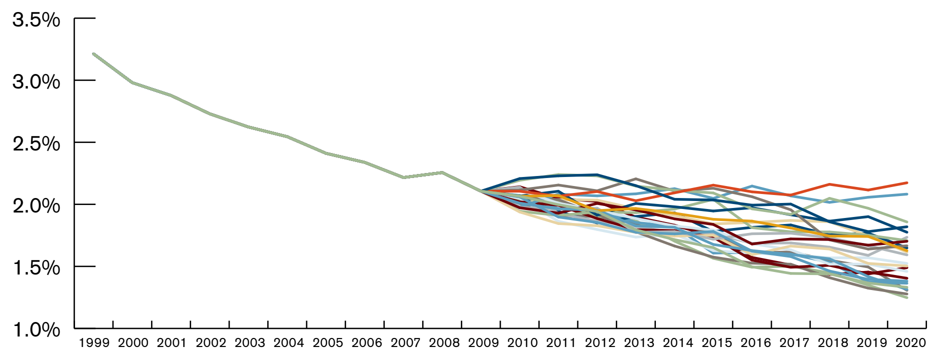
2. **Short-term (annual) mortality improvement volatility**: Concurrent with long-term mortality improvement trends, historical mortality improvement rates fluctuate from year to year. These

fluctuations can be attributed to multiple factors, including extreme weather conditions, new disease strains, or even variations in reporting.

This case study used projected annual mortality improvement volatility, based on volatility parameters captured from historical levels of annual mortality improvement volatility, while ensuring that our long-term mortality improvement volatility target assumptions are also met.

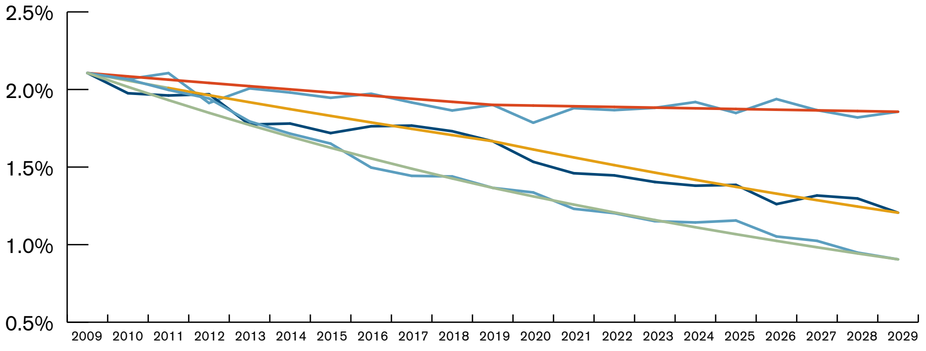
The graph in Figure 2 illustrates the historical general population mortality rate for a 70-year-old male from 1999 to 2009. After 2009, Figure 2 illustrates 25 potential scenarios of the future mortality rate for 70-year-old males, given the average general population mortality improvement rates but reflecting both short-term and long-term volatility exhibited in the historical general population mortality improvement rates.

**FIGURE 2: HISTORICAL AND PROJECTED GENERAL POPULATION ANNUAL MORTALITY RATE (MALE 70 YEARS OLD, 25 SCENARIOS)**




The graph in Figure 3 illustrates the projected 70-year-old mortality rate under three separate potential scenarios. Each scenario was chosen to reflect different levels of mortality improvement over a 20-year period (i.e., less than expected, similar to the expected assumption, and more than expected). There are two lines per scenario. The first line illustrates the mortality rates that vary only based on the long-term trend volatility. The second line illustrates the mortality rates that vary by both long- and short-term volatility. It is worth noting that during the first few years of the projection, the mortality rates–when reflecting the short-term volatility–overlap in each of the three scenarios. However, when projected over time, the long-term patterns emerge. Further, the graph illustrates how the mortality rates reflecting short-term volatility converge to the mortality rates reflecting only long-term volatility at the end of each long-term wave (i.e., in this projection the long-term wave is assumed to be every 10 years).

**FIGURE 3: PROJECTED GENERAL POPULATION MORTALITY RATES BASED ON HISTORICAL ANNUAL AND LONG-TERM MORTALITY IMPROVEMENT VOLATILITY (MALE 70 YEARS OLD, THREE SCENARIOS)**

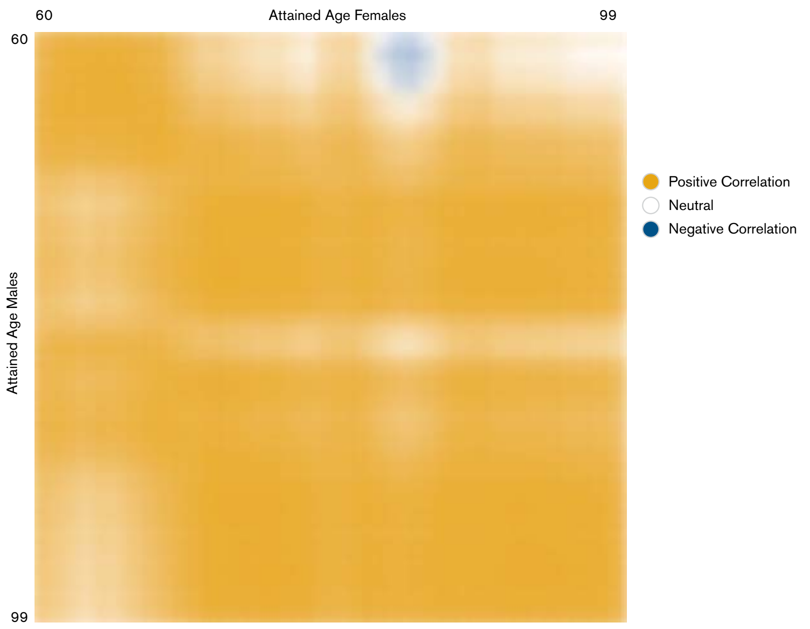



3. **Correlation in mortality improvement trend volatility**: For the purpose of this case study, we analyzed the correlation of annual and long-term mortality improvement across ages and genders. Future mortality improvement rates are stochastically generated such that they are correlated consistently with the observed historical correlation.

The graph in Figure 4 is a heat map of the intensity of the smoothed correlation in long-term mortality improvement between males and females of ages 60 through 99. Areas of blue imply negative correlation. White areas illustrate near-zero correlation. Orange areas imply highly positive correlation. While similar results emerge for same-gender correlations, the heat map in Figure 4 shows a high level of correlation in male and female long-term mortality improvement rates across many age combinations in the UK during 1979-2009. However, there are some combinations that have exhibited low or negative correlation in mortality improvement rates.

**For the purpose of this case study, we analyzed the correlation of annual and long-term mortality improvement across ages and genders.**

**FIGURE 4: HISTORICAL LONG-TERM MORTALITY IMPROVEMENT CORRELATION**



Outside of the trends and volatility of mortality improvement captured above, it is conceivable that events may cause mortality rates to change faster and more abruptly than anticipated in the baseline assumption, even after reflecting mortality improvement trend volatility (derived from historical levels).

### 3. Extreme longevity occurrences

Outside of the trends and volatility of mortality improvement captured above, it is conceivable that events may cause mortality rates to change faster and more abruptly than anticipated in the baseline assumption, even after reflecting mortality improvement trend volatility (derived from historical levels). For example, a medical breakthrough can have a rapid and long-term effect on future death rates related to a specific condition or disease, shifting the mortality curves substantially from their current levels.

As a demonstration within this case study, we consider the contingency of a significant reduction in cancer-related deaths. There were two reasons that we chose to test and simulate potential reductions in cancer-related deaths:

1. As of this writing, there has not been a significant reduction in cancer-related deaths in the period of years used to develop the historical mortality improvement trend volatility statistics.

2. The high level of current medical research may lead to significant advances in treating cancer. However, while there is reason to continue hoping for mortality improvement, we note that past trials have not produced large improvements in aggregate. This is in contrast to the recent history of measurably successful treatments that have reduced deaths from heart disease.

For certain simulations that were designed to evaluate the cost of potential extreme longevity occurrences, we assume a 2% annual probability of a 10% permanent reduction in the mortality rate proportional to those deaths (at each attained age and gender) that are attributable to cancer. Our simulations do not limit the number of possible simulated extreme longevity occurrences (i.e., it is possible for a given scenario to have multiple years with compound reductions in the cancer-related mortality rate). These improvements are in addition to the annual and long-term improvement volatility.

It is important to consider these volatility parameters collectively with the expected baseline mortality improvement assumptions and each of the sources of volatility. It is a matter of individual actuarial judgment to parameterize the volatility assumptions. The volatility assumptions used in this case study are illustrative and should not be considered to be definitive. The actuary has a professional obligation to be satisfied that his or her choice of assumptions is reasonable and supportable.

Note that volatile mortality improvement and extreme longevity occurrences are assumed to be occurring randomly across the entire population each year, and not being generated independently for each life.

## SOLVENCY II VALUATION METHODOLOGY AND RESULTS

To evaluate the sample portfolio under Solvency II requirements, we used the following methodology:

### 1. Best Estimate Liability

The *Best Estimate Liability* value at the Valuation Date is defined as the result of a deterministic projection using the best estimate assumptions for mortality and mortality improvement, and discounted using the spot curve as of December 31, 2009 (risk-free rates with a 100% allowance for the illiquidity premium).

For this case study, we calculated the best estimate cash flows equal to the mean of 2,000 stochastic scenarios where the date of death was randomly determined for each life, consistent with the best estimate assumptions for mortality and mortality improvement with no volatility.[12]

Therefore, our Best Estimate Liability at the Valuation Date was calculated based on the following formulas:

$ia_t$ = Annual Spot Rate from Risk-Free Curve with 100% allowance for illiquidity premium

$BECF_t$ = Average annual annuity payments projected to be paid in year *t* (e.g., best estimate cash flow)

$BEL_0$ = Best Estimate Liability at time zero

$$= \sum_{t=0,1,2,...} \frac{BECF_t}{(1+ia_t)^t}$$

= 1,725.5 million

### 2. Standard Formula Approach

Under Solvency II, the *Standard Formula* may be used to calculate the SCR. The Standard Formula assumes an immediate permanent improvement in mortality rates of 20%. That is, the best estimate mortality assumption is multiplied by 80% in all years. The SCR calculated using the Standard Formula equals the excess of (a) the present values of the cash flows reflecting the 20% margin over (b) the best estimate liability, discounted to the Valuation Date using the spot curves with 100% allowance for the illiquidity premium.

**The Standard Formula assumes an immediate permanent improvement in mortality rates of 20%. That is, the best estimate mortality assumption is multiplied by 80% in all years.**

The SCR using the Standard Formula at the Valuation Date was calculated based on the following formulas:

$SFCF_t$ = Average annual annuity payments projected (using the Standard Formula Mortality Assumption) to be paid in year *t*

*Standard Formula Liability* $= \sum_{t=0,1,2,...} \frac{SFCF_t}{(1+ia_t)^t}$

$SCR_0^{StdForm}$ = *Standard Formula Liability* less $BEL_0$

= 1,884.1 million – 1,725.5 million

= 158.6 million

12 For projections where volatility is limited strictly to the date of death, the stochastic projections converge to the expected mortality (including expected mortality improvement). Because the Best Estimate Liability does not reflect randomization of mortality rates, the results converged within 2,000 scenarios.

QIS5 permits a range of simplifications in the calculation of the Risk Margin. In this case study we employ one of these, and estimate the Risk Margin by amortizing the $SCR^{StdForm}$ over the projection period proportional to the annual annuity cash flows used to determine the Best Estimate Liability:

$$SCR_t^{StdForm} = \text{Amortized SCR at duration } t.$$

$$= SCR_{t-1}^{StdForm} * \frac{BECF_t}{BECF_{t-1}}$$

The Risk Margin is calculated equal to 6.00% (which is a proxy for the cost of capital rate) of the present value of the series of amortized $SCR_t$ values, discounted using the spot curve as of December 31, 2009 (risk-free rates with 0% allowance for the illiquidity premium).

**The Best Estimate Liability plus the Risk Margin plus the SCR will represent the Total Asset Requirement. The excess of the Total Asset Requirement over the Best Estimate Liability will be called the Excess over Best Estimate Liability (ExBEL). This excess will be used as a basis for comparing the Capital Requirement among the three approaches.**

The Best Estimate Liability plus the Risk Margin plus the SCR will represent the Total Asset Requirement under the Standard Formula Approach. The excess of the Total Asset Requirement over the Best Estimate Liability will be called the Excess over Best Estimate Liability (ExBEL). This excess will be used as a basis for comparing the Capital Requirement among the three approaches.

$iz_t$ = Annual Spot Rate from Risk-Free Curve with 0% allowance for illiquidity premium.

$$\text{Risk Margin} = 6\% * \sum_{t=0,1,2,...} \frac{SCR_t^{StdForm}}{(1+iz_t)^t}$$

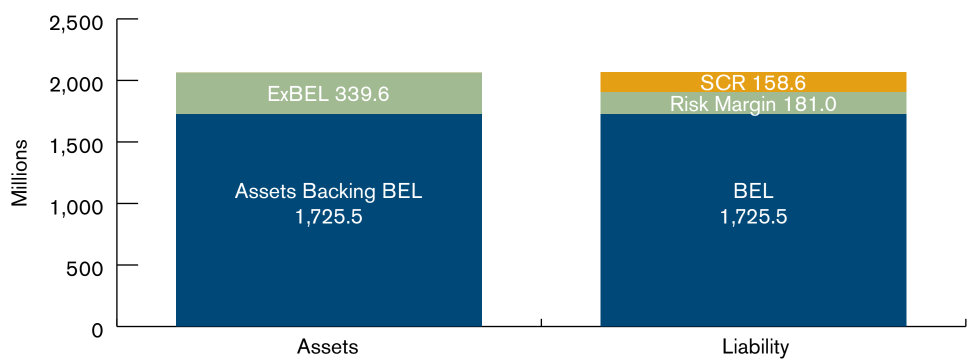
$$= 181.0 \text{ million}$$

$$ExBEL^{StdForm} = \text{Standard Formula SCR} + \text{Standard Formula Risk Margin}$$

$$= 339.6 \text{ million}$$

The chart in Figure 5 illustrates the Total Asset Requirement under the Standard Formula Approach.

**FIGURE 5: STANDARD FORMULA APPROACH RESULTS**



The sum of $SCR^{StdForm}$ and the Risk Margin for this case study equals 339.6 million. In other words, the Total Asset Requirement in excess of the Best Estimate Liability under the Standard Formula Approach is 339.6 million.

### 3. Economic Capital Approach

We now compare the Standard Formula Approach to an Economic Capital calculation utilizing a principles-based approach. In essence, while the Standard Formula applies a seemingly plausible margin (i.e., 20%), it remains to be demonstrated that this margin is appropriate, overly conservative, or not conservative enough. In fact, in QIS5, EIOPA reported, "There was feedback from a number of countries that as the current shock was only a shock on the level, it failed to adequately take into account trend risk: undertakings felt a stress on the future improvement rates would be more appropriate. However, opinion among their supervisors was mixed: some agreed that this shock would be more appropriate, but there were also concerns that this would introduce further complexity to the Standard Formula."[13]

**In essence, while the Standard Formula applies a seemingly plausible margin (i.e., 20%), it remains to be demonstrated that this margin is appropriate, overly conservative, or not conservative enough.**

Utilizing the stochastic projection methodology and volatility parameters discussed above and in Appendix C, we performed 10,000 simulations[14] of (a) future mortality curves for each life in the Hypothetical Portfolio and (b) the resulting future cash flows given the stochastically generated survivorship probabilities. This produced a set of 10,000 simulated aggregate cash flow patterns for the Hypothetical Portfolio.

The cash flows for each scenario were discounted using the spot rate with 100% allowance for the illiquidity premium to determine that scenario's present value. Each of the present values were ranked from highest to lowest and the Total Asset Requirement under the Economic Capital Approach was determined as the 99.5th[15] percentile present value of all the scenarios.

The Economic Capital Approach was performed two ways:

1. **Volatility Assumptions A**: Volatility in the mortality curve was only based on historical mortality improvement trends without reflecting the possibility of a significant reduction in cancer-related deaths.

2. **Volatility Assumptions B**: Volatility in the mortality curve was based on both historical mortality improvement trends and the possibility of a significant reduction in cancer-related deaths.

The Economic Capital Requirement, which is the Total Asset Requirement under the Economic Capital Approach less the Best Estimate Liability (or the ExBEL), at the Valuation Date was calculated based on the following formulas:

$ECCF_t^{scn}$ = Annual annuity payments projected using stochastically generated mortality assumption in scenario *scn* to be paid in year *t*.

$$Liability(scn) = \sum_{t=0,1,2,\ldots} \frac{ECCF_t^{scn}}{(1+ia_t)^t}$$

Total Asset Requirement under Economic Capital Approach = 99.5th percentile value of *Liability(scn)* over all scenarios

13 EIOPA (2011). Report on the fifth Quantitative Impact Study (QIS5) for Solvency II, p.80.
14 Results generally converged within the 10,000 simulations.
15 The 99.5th percentile value was chosen to be consistent with the Solvency II requirements. Each company may have its own view on economic capital requirements.

### Cost of Volatility

An interesting phenomenon occurs when we reflect volatility in our underlying assumptions. When we perform stochastic analysis with static assumptions, the average of the scenario liabilities (over all scenarios) will converge to the deterministic Best Estimate Liability. However, if dynamic assumptions are used instead, the tail percentile values show an asymmetric dispersion, resulting in divergence between (1) the average of the scenario liabilities from the stochastic valuation and (2) the deterministic Best Estimate Liability.

In this case study, the average of the scenario liabilities (over all scenarios) under the two volatility assumptions were 1,726.3 million and 1730.4 million, for Volatility Assumptions A and B, respectively. These results are both higher than the deterministically calculated Best Estimate Liability of 1,725.5 million. The fact that economic liability under the dynamic assumptions is more than that under static assumptions is no coincidence but rather reflects the asymmetry in the annuity payout patterns.

When using symmetric volatility distributions, like Volatility Assumption A, the average beneficiary has an equal chance of living longer than expected or dying sooner than expected. Reflecting volatility increases the range of possible values—both increasing and decreasing values. However, this asymmetry in the present value of cash flows stems from the fact that there is a limited range to how much sooner a beneficiary might die (i.e., on or after the valuation date), but the date to which they might survive is open-ended. Hence, the premature death might eliminate a limited number of annuity payments, but the survivor may receive many years of additional payments.

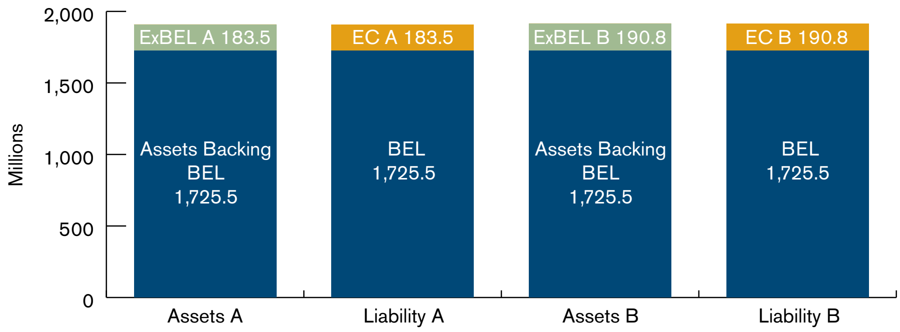
This *Cost of Volatility* is not reflected in the insurer's liability unless mortality volatility is introduced into the equation. However, an insurer investing its capital to issue annuity products should be compensated for this Cost of Volatility.

$ExBEL^{Eco\ Vol\ A}$ = 183.5 million

$ExBEL^{Eco\ Vol\ B}$ = 190.8 million

The chart in Figure 6 illustrates the Total Asset Requirement under the Economic Capital Approach for both volatility assumption sets.

**FIGURE 6: ECONOMIC CAPITAL APPROACH RESULTS**




The Total Asset Requirement in excess of the Best Estimate Liability under the Economic Capital Approach using Volatility Assumptions A is approximately 54% of the Total Asset Requirement in excess of the Best Estimate Liability under the Standard Formula Approach. In this case study the 20% immediate permanent reduction in mortality rates applied in the Standard Formula resulted in a Total Asset Requirement that is more than that required under an Economic Capital Approach reflecting historical levels of volatility and correlation in mortality improvement rates.

The Total Asset Requirement in excess of the Best Estimate Liability under the Economic Capital Approach using Volatility Assumptions B is higher than that for Volatility Assumptions A, but is still significantly less than that implied by the Standard Formula Capital Requirement. Thus, in this case study the Standard Formula resulted in a Total Asset Requirement that is more than that required under an Economic Capital Approach reflecting (a) historical levels of volatility and correlation in mortality improvement rates and (b) a 2% chance in each year that cancer deaths may be permanently reduced by 10%.

While companies hold capital at regulatory required levels, they generally do not set premiums at such a requirement. For illustrative purposes, consider a company that intends to set *Pricing Levels* at one standard deviation from the mean (i.e., similar to the 84th percentile value under a standard normal distribution). The 84th percentile economic liability values in excess of the Best Estimate Liability under Volatility Assumptions A and B are 64.2 million and 68.7 million, respectively. Thus, the initial capital required to issue the business will be much higher than the Pricing Level, which will in turn depress the return on invested capital.

## INTERNAL MODEL APPROACH

Under Solvency II, the SCR is defined as "the potential amount of own funds that would be consumed by unexpected large events whose probability of occurrence within a one-year time frame is 0.5%."[16] In practice, when using an internal model, the projected values may be generated by stochastic formula during the first year, and the best estimate mortality and mortality improvement assumptions in subsequent years must reflect how the prudent actuary might modify his or her prior year's assumptions for this new simulated experience.

Based on our understanding of the Solvency II description of a *regulator-approved internal model*, the stochastic methodology is modified as per the specifications of Solvency II as follows:

The calculation of the economic capital under Solvency II refers to the one-year 99.5% (once-in-200-years event) Value at Risk (VaR) concept. For a specific risk module (the longevity risk in this case study), companies are encouraged to use an internal model, which allows them to compute the SCR for the underlying risk module by using the following formula:

$$\text{Internal Model SCR} = VaR_{99.5\%}\left(\frac{BEL_1}{(1+i(1))} + \sum_{0<t\leq 1}\frac{CF_t}{(1+i(t))^t}\right) - BEL_0$$

- Assume $i(t)$ is the risk-free interest rate with 100% allowance for illiquidity premium.
- $VaR_{99.5\%}(X)$ is the 99.5% percentile of a random variable $X$.
- $BEL_0$ is the best estimate liability at time $t = 0$.
- $CF_t$ is the stochastic cash flow at time $t$ in each scenario (i.e., annuity benefits paid).
- $BEL_1$ is the best estimate liability at time $t = 1$, following the underlying scenario between $t = 0$ and $t = 1$. For purposes of this case study, we estimate this value based on the same formula for $BEL_0$ described above, except with an altered mortality expectation that reflects the simulated experience between the Valuation Date and the end of the first projection year. In other words, when determining the best estimate assumptions one year after the Valuation Date, we expect the actuary will consider its original expectations on the Valuation Date as well as the simulated experience that occurred in the scenario from the Valuation Date to the end of the first projection year.

Current levels of mortality rates at $t = 0$ (to calculate $BEL_1$ ) reflect the lingering effect of simulated mortality improvement during the first projection year. For each scenario, we assume the modified level of expected mortality improvements after first projection year is a weighted average of the stochastic first-year mortality improvement and the expected level of mortality improvement:

| | |
|---|---|
| $\Delta q_{x,t}$ | = Expected annual rate of mortality improvement at attained age $x$, duration $t$ |
| $Q_x^{scale}(0)$ | = Stochastic adjustment to mortality improvement[17] of attained age $x$, duration 0 |

**For purposes of this case study, we estimate this value based on the same formula for $BEL_0$ described above, except with an altered mortality expectation that reflects the simulated experience between the Valuation Date and the end of the first projection year. In other words, when determining the best estimate assumptions one year after the Valuation Date, we expect the actuary will consider its original expectations on the Valuation Date as well as the simulated experience that occurred in the scenario from the Valuation Date to the end of the first projection year.**

16 EIOPA (2011). Report on the fifth Quantitative Impact Study (QIS5) for Solvency II, p. 23.
17 See Appendix C: Stochastic Modelling.

| | |
|---|---|
| $\Delta q_{x,t}^{new}$ | = Expected annual rate of mortality improvement at attained age $x$, duration $t$ reflecting one-year stochastic volatility |
| | $= 1 - (1 - \Delta q_{x,t}) * Q_x^{scale}(0)$ |
| $c$ | = Credibility assigned to one-year stochastic mortality improvement in subsequent years |
| $\Delta q_{x,t}^{Sol2}$ | = Expected annual rate of mortality improvement at attained age $x$, duration $t$ reflecting the original expectation and one-year stochastic volatility |
| | $= (1 - c) * \Delta q_{x,t} + c * \Delta q_{x,t}^{new}$ |

where the parameter $c$ represents a credibility factor applied to the realized mortality improvement at time $t = 1$. When $c$ is set to zero, the future mortality improvement assumption is equal to the original best estimate expectation at the Valuation Date. When $c$ is set to 100%, the future mortality improvement assumption is entirely based on the simulated experience in the first projection year. For purpose of this case study, we assume $c = 10\%$[18].

**The methodology reflects stochastic annual mortality improvements for the first year, and then, for each scenario, assuming the mortality improvement after the first year equals the expected mortality improvement, adjusted for the ratio of change of the first year random mortality improvement factor over expected, given a credibility factor of 10%.**

When determining the present value of the scenario cash flows at $t = 1$, the methodology reflects stochastic annual mortality improvement for the first year, and then, for each scenario, assuming the mortality improvement after the first year equals the expected mortality improvement, adjusted for the ratio of change of the first-year random mortality improvement factor[19] over expected, given a credibility factor of 10%.

The Internal Model Risk Margin is calculated in a similar manner to the Risk Margin described in the Standard Formula, where the only difference is the starting SCR.

We determined the Total Asset Requirement under the Internal Model Approach under both Volatility Assumptions A and B. The SCR is described above. The Risk Margin is calculated similarly to that under the Standard Formula, but amortizing the SCR that was calculated under the Internal Model Approach.

| | |
|---|---|
| Internal Model $SCR_A$ | = 140.5 million |
| Internal Model Risk $Margin_A$ | = 160.3 million |
| Excess over Best Estimate Liability Under Internal Model Approach (A) | = Internal Model $SCR_A$ + Internal Model Risk $Margin_A$ |
| | = 300.8 million |
| Internal Model $SCR_B$ | = 140.9 million |

18 We selected the credibility percentage $c = 10\%$ to reflect the amount of credibility an actuary might plausibly give to one year's experience. The actuary should apply his or her professional judgment regarding the level of credibility given to the year's experience. In this case, given that it is only one year's experience, we would expect the actuary to limit its effect. The actuary's assumptions may vary depending on the specifics of the situation, the company's internal valuation practice, and the actuary's individual judgment.

19 In this case study, we assume that the revised expectation in mortality improvement is based solely on historical mortality improvement volatility.

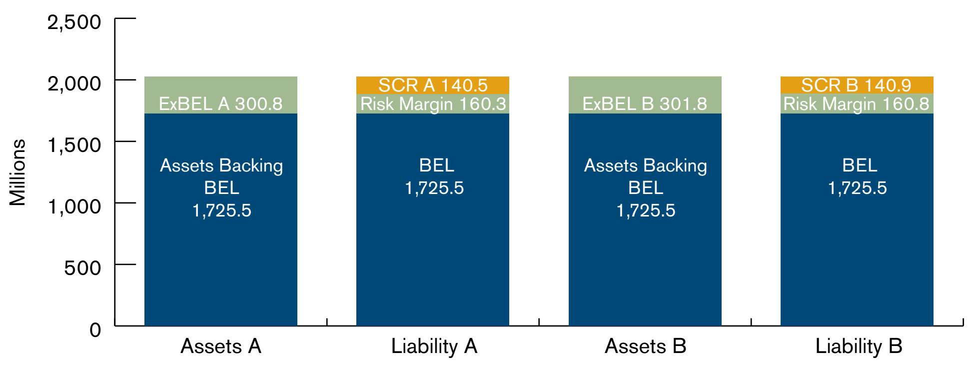
Internal Model Risk Margin$_B$ = 160.8 million

Excess over Best Estimate Liability Under Internal Model Approach (B) = Internal Model SCR$_B$ + Internal Model Risk Margin$_B$

= 301.8 million

The chart in Figure 7 illustrates the Total Asset Requirement under the Internal Model Approach for both volatility assumptions sets.

**FIGURE 7: INTERNAL MODEL APPROACH RESULTS**




According to this case study, the Total Asset Requirement when using an Internal Model is significantly less than the Total Asset Requirement when using the Standard Formula. As compared to the Standard Formula, the Excess over Best Estimate Liability is respectively reduced by 38.8 million (approximately 11.4%) and 37.8 million (approximately 11.1%) under Volatility Assumptions A and B.

**According to this case study, the Total Asset Requirement when using an Internal Model is significantly less than the Total Asset Requirement when using the Standard Formula.**

While there are capital savings by using an Internal Model (relative to the Standard Formula), in this case study the Total Asset Requirement when using an Internal Model is still higher than that under an Economic Capital Approach. If a company can receive regulatory approval to use an Economic Capital Approach as its Internal Model to determine the Total Asset Requirement, it may achieve a higher return on its invested capital. Otherwise, companies may benefit by entering into financial transactions that move the longevity risk to regulatory environments that are more favorable.

Given the dramatic disparity in the Total Asset Requirements, we feel it is important to highlight the main difference driving these results. The definition of the Internal Model requirements examines potential changes in the Best Estimate Liability assumptions one year from the Valuation Date (recalibrated based on experience simulated during the first projection year). In contrast, the Standard Formula requires a shift in the mortality curve for all future years. At least 9,950 of the 10,000 stochastic scenarios generated by the Internal Model did not produce as significant a shift in mortality rates.

It is informative to compare the Internal Model Approach to the Economic Capital Approach. The Internal Model SCR captures a little more than one year's volatility (i.e., volatility from the Valuation Date to the beginning of the second projection year and 10% credibility on the simulated first projection year's mortality improvement to determine future expected mortality improvement). The Internal Model Risk Margin can be considered a proxy for the cost of uncertainty after the first year. In contrast, the Economic Capital Requirement directly reflects volatility in all future projection years. In

**If a company can receive regulatory approval to use an Economic Capital Approach as its internal model to determine the Total Asset Requirement, it may achieve a higher return on its invested capital. Otherwise, companies may benefit by entering into financial transactions that move the longevity risk to regulatory environments that are more favorable.**

this case study, the Total Asset Requirement under the Internal Model Approach is more than that of the Economic Capital Approach (under both volatility assumptions sets).

It is important to note that the Internal Model Approach employed in this case study implies the actuary will apply full credibility to the simulated experience over the first projection year when determining a best estimate mortality table as of the end of the first projection year. This is in contrast to our further assumption that the actuary will apply only 10% credibility to the simulated experience over the first projection year when determining a future mortality improvement assumption. That is, our approach assumes the actuary will use the most recent mortality experience to develop the current mortality table, but will only give partial credibility to the most recent experience when developing a future *improvement* assumption. It is conceivable that an actuary may not use a mortality table at the end of the first projection year that fully reflects the past (single) year's mortality experience. If the Internal Model Approach applied a lower credibility than 100% to the most recent year's mortality experience when developing the baseline mortality table at the end of the first projection year, it would have resulted in a lower Total Asset Requirement because the extreme scenarios determining the 1-in-200-year event would be moderated toward prior expectations with respect to that baseline mortality table.

It is also informative to compare the Internal Model Approach under both volatility assumptions sets. In effect, Set A provides the foundation onto which Set B adds a reduction to cancer deaths in 2% of the scenarios. Such scenarios with added improvement from cancer breakthroughs rise up the ranked order of present values, but the effect on the Total Asset Requirement is determined by the 50th highest value. Only one scenario that was not in the top 50 Set A scenarios (i.e., with the highest present values using Volatility Assumptions A) moved into the top 50 Set B scenarios (i.e., with the highest present values using Volatility Assumptions B). This caused a slight shift of the 99.5th percentile scenario and a slightly higher Total Asset Requirement; however, the determination of the Total Asset Requirement under the Internal Model Approach was primarily driven by the annual volatility in mortality improvement. The volatility that was due to the possibility of a significant reduction in cancer-related deaths had more of an impact on the Total Asset Requirement under the Economic Capital Approach (than under the Internal Model Approach) because the Economic Capital Approach recognizes this possibility occurring in any year, not just the first projection year.

## CONCLUSION

The charts in Figures 8 and 9 summarize the values (in millions) discussed in the sections above.

FIGURE 8: COMPONENTS OF TOTAL ASSET REQUIREMENT (TAR)

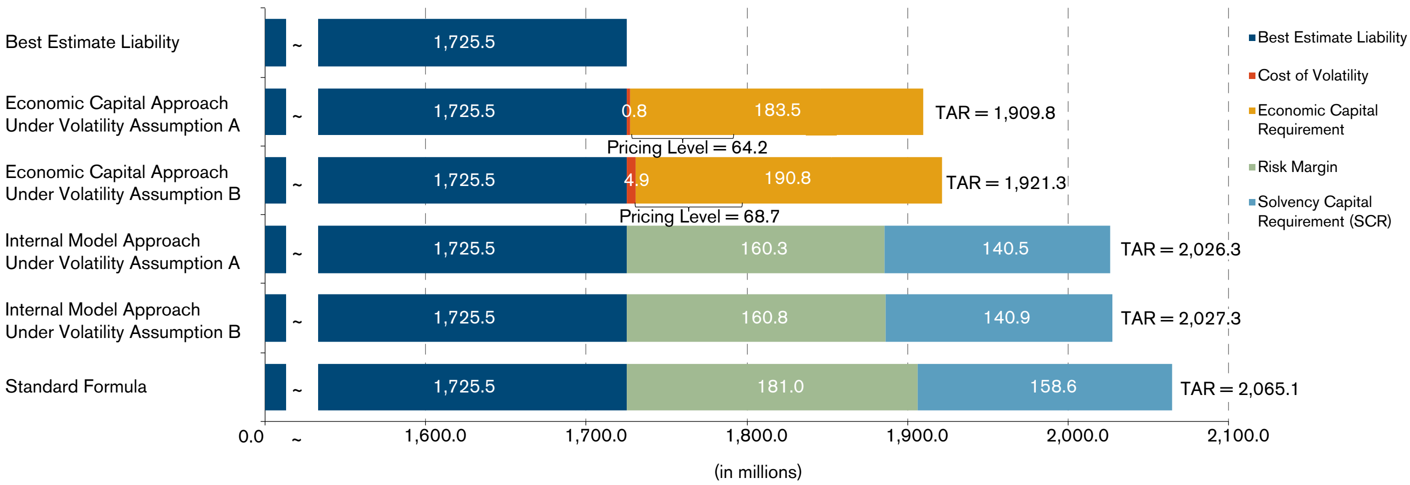
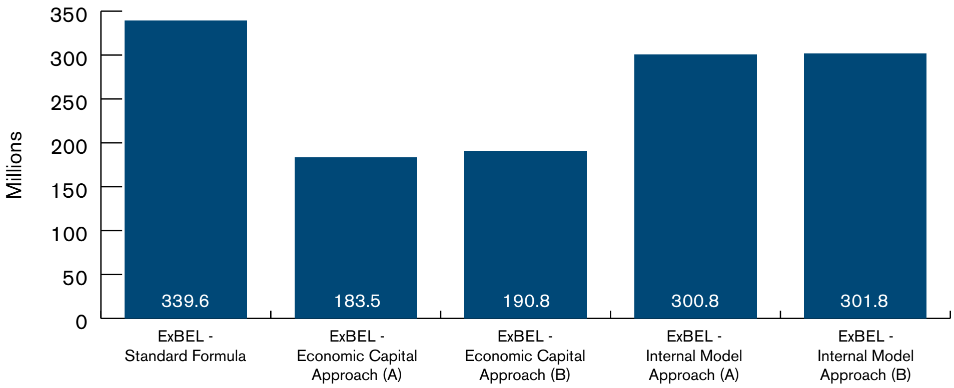



FIGURE 9: SUMMARY RESULTS – EXCESS OVER BEST ESTIMATE LIABILITY



**Even though there are still capital savings relative to the Standard Formula Approach, the Internal Model Approach resulted in higher capital requirements than a principles-based Economic Approach.**

The following list contains a few relevant observations specific to this case study.

- Utilizing the Standard Formula Approach results in a higher Total Asset Requirement compared to a Total Asset Requirement under an Economic Capital Approach that is a stochastic analysis reflecting (a) historical levels of volatility and correlation, and (b) the possibility of extreme longevity occurrences.

- Utilizing an Internal Model that stochastically generates future mortality curves based on (a) historical levels of volatility and correlation, and (b) the possibility of extreme longevity occurrences, may result in companies having a Total Asset Requirement that is less than required under the Standard Formula.

- Even though there are still capital savings relative to the Standard Formula Approach, the Internal Model Approach resulted in higher capital requirements than a principles-based Economic Approach.

It is important to note that this case study was performed on a hypothetical portfolio of lives using the assumptions described above. It is conceivable that applying these techniques to a real portfolio or utilizing different expected and volatility assumptions may lead to different conclusions. Each actuary should use his or her own judgment when developing expected and volatility assumptions.

# APPENDIX A

## Hypothetical portfolio characteristics

**MEASURING LIFE**

| | BY COUNT | | BY AMOUNT | |
|---|---|---|---|---|
| | VALUE | PCT | VALUE | PCT |
| PRIMARY ANNUITANT | 40,015 | 80% | 63,427,670 | 67% |
| SPOUSE (WIDOW/WIDOWER) | 9,985 | 20% | 31,176,850 | 33% |

**GENDER**

**PRIMARY ANNUITANT**

| | BY COUNT | | BY AMOUNT | |
|---|---|---|---|---|
| | VALUE | PCT | VALUE | PCT |
| MALE | 26,505 | 53% | 49,600,830 | 52% |
| FEMALE | 23,495 | 47% | 45,003,690 | 48% |

**MEASURING LIFE**

| | BY COUNT | | BY AMOUNT | |
|---|---|---|---|---|
| | VALUE | PCT | VALUE | PCT |
| MALE | 27,500 | 55% | 52,057,080 | 55% |
| FEMALE | 22,500 | 45% | 42,547,440 | 45% |

## JOINT LIFE (OR SURVIVORSHIP) BENEFITS

### BY COUNT

| MEASURING LIFE AS GROUP | JOINT/SURVIVOR | SINGLE | TOTAL | PCT |
|---|---|---|---|---|
| 55-59 | - | - | - | |
| 60-64 | 6,195 | 680 | 6,875 | 90% |
| 65-69 | 9,520 | 1,445 | 10,965 | 87% |
| 70-74 | 8,735 | 1,800 | 10,535 | 83% |
| 75-79 | 7,420 | 2,080 | 9,500 | 78% |
| 80-84 | 4,910 | 2,135 | 7,045 | 70% |
| 85-89 | 1,955 | 1,590 | 3,545 | 55% |
| 90-94 | 405 | 640 | 1,045 | 39% |
| 95-99 | 95 | 375 | 470 | 20% |
| 100+ | - | 20 | 20 | 0% |

### BY VALUE

| MEASURING LIFE AS GROUP | JOINT/SURVIVOR | SINGLE | TOTAL | PCT |
|---|---|---|---|---|
| 55-59 | - | - | - | |
| 60-64 | 11,696,685 | 1,025,055 | 12,721,740 | 92% |
| 65-69 | 18,868,405 | 2,310,530 | 21,178,935 | 89% |
| 70-74 | 16,968,975 | 3,149,585 | 20,118,560 | 84% |
| 75-79 | 14,228,160 | 3,739,660 | 17,967,820 | 79% |
| 80-84 | 9,996,950 | 3,446,145 | 13,443,095 | 74% |
| 85-89 | 4,000,700 | 2,560,595 | 6,561,295 | 61% |
| 90-94 | 838,685 | 964,730 | 1,803,415 | 47% |
| 95-99 | 225,730 | 548,610 | 774,340 | 29% |
| 100+ | - | 35,320 | 35,320 | 0% |

## ADJUSTED BENEFITS

| | BY COUNT | | BY AMOUNT | |
|---|---|---|---|---|
| | VALUE | PCT | VALUE | PCT |
| INDEXED | 42,525 | 85% | 79,788,090 | 84% |
| NONE | 7,475 | 15% | 14,816,430 | 16% |

## AGE AT VALUATION DATE

### PRIMARY ANNUITANT

| | BY COUNT | | BY AMOUNT | |
|---|---|---|---|---|
| PRIMARY LIFE AGE GROUP | VALUE | PCT | VALUE | PCT |
| 54-59 | 440 | 1% | 1,338,470 | 1% |
| 60-64 | 6,740 | 13% | 12,574,380 | 13% |
| 65-69 | 10,740 | 21% | 19,957,255 | 21% |
| 70-74 | 10,495 | 21% | 20,044,920 | 21% |
| 75-79 | 9,350 | 19% | 17,951,820 | 19% |
| 80-84 | 7,065 | 14% | 13,239,405 | 14% |
| 85-89 | 3,500 | 7% | 6,362,465 | 7% |
| 90-94 | 1,160 | 2% | 2,269,985 | 2% |
| 95-99 | 465 | 1% | 789,740 | 1% |
| 100+ | 45 | 0% | 76,080 | 0% |

### SPOUSE

| | BY COUNT | | BY AMOUNT | |
|---|---|---|---|---|
| SECONDARY LIFE AGE GROUP | VALUE | PCT | VALUE | PCT |
| 54-59 | 1,520 | 4% | 2,523,030 | 3% |
| 60-64 | 5,720 | 15% | 11,127,655 | 14% |
| 65-69 | 8,695 | 22% | 17,588,010 | 23% |
| 70-74 | 8,565 | 22% | 16,434,425 | 21% |
| 75-79 | 7,065 | 18% | 13,628,840 | 18% |
| 80-84 | 4,590 | 12% | 9,469,880 | 12% |
| 85-89 | 2,340 | 6% | 4,495,010 | 6% |
| 90-94 | 605 | 2% | 1,257,090 | 2% |
| 95-99 | 135 | 0% | 300,350 | 0% |
| 100+ | - | 0% | - | 0% |

### MEASURING LIFE

| | BY COUNT | | BY AMOUNT | |
|---|---|---|---|---|
| MEASURING LIFE AGE GROUP | VALUE | PCT | VALUE | PCT |
| 54-59 | - | 0% | - | 0% |
| 60-64 | 6,875 | 14% | 12,721,740 | 13% |
| 65-69 | 10,965 | 22% | 21,178,935 | 22% |
| 70-74 | 10,535 | 21% | 20,118,560 | 21% |
| 75-79 | 9,500 | 19% | 17,967,820 | 19% |
| 80-84 | 7,045 | 14% | 13,443,095 | 14% |
| 85-89 | 3,545 | 7% | 6,561,295 | 7% |
| 90-94 | 1,045 | 2% | 1,803,415 | 2% |
| 95-99 | 470 | 1% | 774,340 | 1% |
| 100+ | 20 | 0% | 35,320 | 0% |

| ANNUALIZED ANNUITY AMOUNT | | | | |
|---|---|---|---|---|
| | BY COUNT | | BY AMOUNT | |
| ANNUALIZED AMOUNT | VALUE | PCT | VALUE | PCT |
| <1K | 28,990 | 58% | 21,686,315 | 23% |
| 1K-5K | 17,695 | 35% | 42,513,475 | 45% |
| 5K-10K | 2,775 | 6% | 20,832,970 | 22% |
| 10K-20K | 450 | 1% | 6,760,300 | 7% |
| 20K-30K | 50 | 0% | 1,294,135 | 1% |
| 30K+ | 40 | 0% | 1,517,325 | 2% |

# APPENDIX B

## Discount interest rates

| SPOT RATES | | |
|---|---|---|
| DURATION FROM VALUATION | 0.0% ILLIQUIDITY PREMIUM | 100.0% ILLIQUIDITY PREMIUM |
| 1 | 0.92% | 1.74% |
| 2 | 1.88% | 2.70% |
| 3 | 2.60% | 3.42% |
| 4 | 3.07% | 3.89% |
| 5 | 3.39% | 4.21% |
| 6 | 3.61% | 4.43% |
| 7 | 3.79% | 4.61% |
| 8 | 3.94% | 4.76% |
| 9 | 4.06% | 4.88% |
| 10 | 4.17% | 4.99% |
| 11 | 4.25% | 5.07% |
| 12 | 4.33% | 5.15% |
| 13 | 4.38% | 5.20% |
| 14 | 4.42% | 5.24% |
| 15 | 4.45% | 5.27% |
| 16 | 4.47% | 5.29% |
| 17 | 4.48% | 5.30% |
| 18 | 4.47% | 5.29% |
| 19 | 4.46% | 5.28% |
| 20 | 4.45% | 5.27% |
| 21 | 4.42% | 5.24% |
| 22 | 4.39% | 5.21% |
| 23 | 4.36% | 5.18% |
| 24 | 4.33% | 5.15% |
| 25 | 4.30% | 5.12% |
| 26 | 4.26% | 5.08% |
| 27 | 4.23% | 5.05% |
| 28 | 4.20% | 5.02% |
| 29 | 4.17% | 4.99% |
| 30 | 4.14% | 4.96% |
| 31 | 4.12% | 4.77% |
| 32 | 4.09% | 4.58% |
| 33 | 4.07% | 4.40% |
| 34 | 4.05% | 4.22% |
| 35 | 4.03% | 4.03% |
| 36 | 4.02% | 4.02% |
| 37 | 4.01% | 4.01% |
| 38 | 3.99% | 3.99% |
| 39 | 3.98% | 3.98% |
| 40 | 3.97% | 3.97% |
| 41 | 3.97% | 3.97% |
| 42 | 3.96% | 3.96% |
| 43 | 3.95% | 3.95% |
| 44 | 3.95% | 3.95% |
| 45 | 3.94% | 3.94% |
| 46 | 3.94% | 3.94% |

| SPOT RATES (CONTINUED) | | |
|---|---|---|
| DURATION FROM VALUATION | 0.0% ILLIQUIDITY PREMIUM | 100.0% ILLIQUIDITY PREMIUM |
| 47 | 3.93% | 3.93% |
| 48 | 3.93% | 3.93% |
| 49 | 3.92% | 3.92% |
| 50 | 3.92% | 3.92% |
| 51 | 3.92% | 3.92% |
| 52 | 3.92% | 3.92% |
| 53 | 3.92% | 3.92% |
| 54 | 3.92% | 3.92% |
| 55 | 3.92% | 3.92% |
| 56 | 3.92% | 3.92% |
| 57 | 3.92% | 3.92% |
| 58 | 3.92% | 3.92% |
| 59 | 3.92% | 3.92% |
| 60 | 3.92% | 3.92% |
| 61 | 3.93% | 3.93% |
| 62 | 3.93% | 3.93% |
| 63 | 3.93% | 3.93% |
| 64 | 3.93% | 3.93% |
| 65 | 3.94% | 3.94% |
| 66 | 3.94% | 3.94% |
| 67 | 3.94% | 3.94% |
| 68 | 3.94% | 3.94% |
| 69 | 3.95% | 3.95% |
| 70 | 3.95% | 3.95% |
| 71 | 3.95% | 3.95% |
| 72 | 3.96% | 3.96% |
| 73 | 3.96% | 3.96% |
| 74 | 3.96% | 3.96% |
| 75 | 3.96% | 3.96% |
| 76 | 3.97% | 3.97% |
| 77 | 3.97% | 3.97% |
| 78 | 3.97% | 3.97% |
| 79 | 3.97% | 3.97% |
| 80 | 3.98% | 3.98% |
| 81 | 3.98% | 3.98% |
| 82 | 3.98% | 3.98% |
| 83 | 3.98% | 3.98% |
| 84 | 3.99% | 3.99% |
| 85 | 3.99% | 3.99% |
| 86 | 3.99% | 3.99% |
| 87 | 3.99% | 3.99% |
| 88 | 4.00% | 4.00% |
| 89 | 4.00% | 4.00% |
| 90 | 4.00% | 4.00% |
| 91 | 4.00% | 4.00% |
| 92 | 4.00% | 4.00% |
| 93 | 4.01% | 4.01% |
| 94 | 4.01% | 4.01% |
| 95 | 4.01% | 4.01% |
| 96 | 4.01% | 4.01% |

## SPOT RATES (CONTINUED)

| DURATION FROM VALUATION | 0.0% ILLIQUIDITY PREMIUM | 100.0% ILLIQUIDITY PREMIUM |
|---|---|---|
| 97 | 4.01% | 4.01% |
| 98 | 4.02% | 4.02% |
| 99 | 4.02% | 4.02% |
| 100 | 4.02% | 4.02% |
| 101 | 4.02% | 4.02% |
| 102 | 4.02% | 4.02% |
| 103 | 4.03% | 4.03% |
| 104 | 4.03% | 4.03% |
| 105 | 4.03% | 4.03% |
| 106 | 4.03% | 4.03% |
| 107 | 4.03% | 4.03% |
| 108 | 4.03% | 4.03% |
| 109 | 4.03% | 4.03% |
| 110 | 4.04% | 4.04% |
| 111 | 4.04% | 4.04% |
| 112 | 4.04% | 4.04% |
| 113 | 4.04% | 4.04% |
| 114 | 4.04% | 4.04% |
| 115 | 4.04% | 4.04% |
| 116 | 4.04% | 4.04% |
| 117 | 4.05% | 4.05% |
| 118 | 4.05% | 4.05% |
| 119 | 4.05% | 4.05% |
| 120 | 4.05% | 4.05% |

# APPENDIX C

## Stochastic modelling

This appendix contains a detailed description of the stochastic projection methodology and volatility parameters. The stochastic projections reflect three sources of volatility:

1. Randomized dates of death
2. Future mortality improvement trends volatility
3. Potential extreme longevity occurrences (in excess of historical mortality improvement trend volatility)

### 1. Randomized dates of death

The date of death for each life in the population is determined by a Monte Carlo simulation with an expected result consistent with the expected mortality rates for the given life within that scenario after stochastically generating the mortality curve (described below). Specifically, a random date of death for each participant is calculated by generating uniform random numbers and testing against ${}_tp_x$ for each life *x*.

For each scenario, a random number, *u* (uniformly distributed between 0 and 1), is generated for each life. At time *t*, the random number is compared to the cumulative survival factor (${}_tp_x$) at that duration:

- If $u \le {}_tp_x$ then the participant is still alive.
- Naturally, ${}_tp_x$ is assumed to diminish to zero with increasing *t*. Therefore, at the earliest *t* at which $u > {}_tp_x$, the date of death is fixed to the current date (corresponding to the time *t*).

### 2. Future mortality improvement trend volatility

#### *Introduction*

For ages 60 to 99, we model stochastic mortality improvement reflecting two mortality improvement risk components, a long-term factor reflecting the mortality improvement over *T* years (where we assume $T = 10$), and an annual factor reflecting the year-by-year mortality improvement. Volatility parameters were developed from historical population mortality data.

This section will describe the calculation of the random improvement $M_{x,t}^{new}$, which is linked to the $\delta_{x,t}$ parameter through the following formulas:

$\Delta q_{x,t}$ = Expected annual rate of mortality improvement at attained age *x*, duration $t = 0,1,2,\ldots$

$\delta_{x,t}$ = Stochastic adjustment to mortality improvement of attained age *x*, duration *t*

$\Delta q_{x,t}^{new}$ = Annual rate of mortality improvement reflecting stochastic volatility, at attained age *x*, duration *t*.

$$= 1 - (1 - \Delta q_{x,t} + \delta_{x,t})$$

#### *Long-term improvement factor*

The observation that mortality rates have tended to improve over time has been attributed to a variety of causes (e.g., new medical technology, new drugs and treatments, and standards of hygiene and nutrition). But the long-term trends are often obscured by short-term volatility. For example, the historical mortality improvement in one year is often offset by the next year's improvement. Nevertheless, we can observe long-term trends in improvement of mortality by measuring change in mortality rates over several years, effectively removing the short-term static. This effect can be visualized as waves of mortality improvement. We reflect these long-term trends in mortality improvements by fitting a stochastic model to the historical population mortality data.

Furthermore, it is important to recognize the statistical correlation between the long-term improvement rates for different ages and genders. That is, the causes of mortality improvement will typically not affect a single age or age group but have a broader impact across the population. The correlation is reflected in the stochastic model based on historical population mortality data.

Let us consider an age $x$ (or an age group denoted by $x$) and $N$ years of historical UK population mortality data (e.g., $N = 30$: 1979–2009). The random annualized mortality improvement factor for each $T$-year period will be represented by the random variable, $W^T_{x,t}$, assumed to follow a normal distribution.

$$W^T_{x,t} \sim N(M_x, (\sigma_x)^2)$$

This may also be expressed as:

$$W^T_{x,t} = M_x + \sigma_x \times \varepsilon^x_t$$

$\varepsilon^x_t$ is defined as correlated standard normal random variables, simulated over ages and genders and correlated using a Cholesky decomposition of the empirical (historical) correlation matrix. In this case, the correlation structure is a matrix of correlations among ages and genders:

$$\rho_{x,y} = (Corr(\varepsilon^x_t, \varepsilon^y_t))_{x,y}\text{ , for all sex/age combinations } x \text{ and } y\text{, and for } t = 1,2,\ldots$$

This method requires that the empirical correlation matrix has some algebraic properties guaranteeing the existence of the Cholesky matrix $C$. In the case that the empirical correlation matrix fails to verify the sufficiency conditions (usually being positive semidefinite), the correlation matrix may be approximated using various alternative techniques (e.g., hypersphere optimization, spectral approach,[20] and Higham algorithm[21]).

The values $\rho_{x,y}$ capture the correlations of average improvement for ages $x$ and $y$ over three consecutive 10-year historical improvements (i.e., 1979-1989, 1989-1999, and 1999-2009).

$M_x$ is the average of 10-year improvement factors over the entire period (i.e., 1979-2009) for each age (or age group) $x$.

$\sigma_x$ is the standard deviation of average annualized mortality improvement factors over each of the consecutive $T$-year periods within the $N$ years of historical experience (i.e., three 10-year periods over 30 years: 1979-1989, 1989-1999, and 1999-2009).

For each age group $x$, the random variable $W^T_{x,t}$ will be sampled every $T$-year period in the projection.

***Annual mortality improvement factor***

In addition to the long-term mortality improvement, data shows significant annual mortality improvement fluctuation. The stochastic projections takes this effect into account in a fashion consistent with that used for long-term mortality improvement. Mortality improvement rates are

20 Rebonato, R. (1999). The Most General Methodology to Create a Valid Correlation Matrix for Risk Management and Option Pricing Purposes. Quantitative Research Centre of the NatWest Group.

21 Higham, N. (2003). A Semidefinite Programming Approach for the Nearest Correlation Matrix Problem. University of Waterloo.

projected such that they oscillate around the random annualized long-term improvement factor for each $T$-year period.

Let $m_t$ the expected annual mortality improvement factor for each age (or age group) and $T$-year period, equal the random variable for long-term volatility $W_{x,t}^{T}$.

Define $M_{x,t}^{*}$ as an intermediate random variable representing *non-adjusted* random annual improvement for a one-year period within a given $T$-year projection period. This random variable follows a normal distribution:

$$M_{x,t}^{*} \sim N(m_t, (\sigma_x^1)^2)$$

This may also be expressed as:

$$M_{x,t}^{*} = W_{x,t}^{T} + \sigma_x^1 \times \varepsilon_t^x$$

$\varepsilon_t^x$ are correlated standard normal random variables.

$\sigma_x^1$ is the average of standard deviations of annual mortality improvement rates within each of the consecutive $T$-year periods contained in the $N$ years of historical data (e.g., the three 10-year periods).

Similar to the long-term mortality improvement factors, annual mortality improvement factors are determined by correlating $\varepsilon_t^x$ (by age and gender) using Cholesky factors such that the final annual improvement factors ($M_{x,t}^{new}$ described below) have the empirical annual correlation from 1979 to 2009.

The annual mortality improvement factors ($M_{x,t}^{*}$) are adjusted such that their geometric average over each projected $T$-year period $(\prod_{s=1}^{T} M_{x,s}^{*})^{1/T}$ equals the stochastically generated long-term improvement for that period, $W_{x,t}^{T}$, as follows:

$$M_{x,t}^{new} = W_{x,t}^{T} * \frac{M_{x,t}^{*}}{(\prod_{s=1}^{T} M_{x,s}^{*})^{1/T}}$$

And the difference between the adjusted stochastic mortality improvement factors and the historical is captured as the stochastic adjustment:

$$\delta_{x,t} = M_{x,t}^{new} - M_x$$

### *Scalar for stochastic improvement*

The expected mortality including expected improvement will be modified during the projection to reflect stochastic mortality improvement by application of a scalar $Q_x^{scale}(Dur)$:

$$Q_x^{scale}(Dur) = \prod_{t=1}^{Dur} \frac{((1 - \Delta q_{x+Dur-1,t}) + \delta_{x+Dur-1,t})}{(1 - \Delta q_{x+Dur-1,t})}$$

### 3. Cause of death

To model stochastic mortality by cause of death (COD), a uniformly distributed random number between 0 and 1, $X_t^{COD}$ is generated each projection year $t$ for each cause-of-death category (i.e., for purpose of this case study, we test neoplasm-related causes of death only). If the random variable is less than a probability factor for that category, $P^{COD}$, then the mortality in that year and in all future years is adjusted to reflect a reduction in death rates attributed to that cause equal to the magnitude

$Y^{COD}$ (for example, this case study uses the assumptions 2% annual probability of a 10% reduction caused by neoplasm).

The random adjustment factor for specific cause of death is derived for all durations according to the following recursive formulas:

Let: $D_{x,s}^{COD}$ = The percentage of reported deaths attributable to COD, at attained age $x$, gender $s$, derived from historical UK population data for selected years (e.g., this case study used the period 2004–2009)

$X_t^{COD}$ = A uniformly distributed random number between 0 and 1, for each duration $t = 1,2,\ldots$ and COD

$P^{COD}$ = User-selected annual probability (0%–100%) of an event relative to COD

$Y^{COD}$ = User-selected percentage reduction to deaths that are due to cause COD in case of the simulated probabilistic event

$$O_t^{COD} = \begin{cases} 1 \text{ if } X_t^{COD} < P^{COD}, \text{ for each COD and } t = 1,2,\ldots \\ 0, \text{ otherwise} \end{cases}$$

Then: $Adj_{x,t,s}^{COD}$ = Scalar applied to annual mortality to reflect stochastic volatility by COD, for attained age ($x$), duration ($t = 0,1,\ldots$), and gender $s$ = male or female

$$= \begin{cases} D_{x,s}^{COD}, & t = 0 \\ Adj_{x,t-1,s}^{COD} * (1 - Y^{COD} * O_t^{COD}), & t = 1,2,\ldots \end{cases}$$

$Adj_{x,t,s}$ = Scalar applied to annual mortality to reflect stochastic volatility by all COD, for attained age ($x$), duration ($t$), and gender ($s$)

$$= \sum_{COD} Adj_{x,t,s}^{COD}$$

This reflects the reduction in each cause-of-death contribution to the total mortality rate when the sampled random number is less than the user-specified probability. The $Adj_{x,t,s}$ is applied to mortality rates prior to adjustment to calculate the resulting mortality rate adjusted for simulated reductions from cause of deaths.

The resulting stochastic adjustment factors for mortality improvement volatility ($Q_x^{scale}(t)$) and cause-of-death volatility ($Adj_{x,t,s}$) are applied to the best estimate mortality ($Q_x^{exp}$) as follows:

$$Q_{x,t}^{stoch} = Q_{x,t}^{exp} * Q_x^{scale}(t) * Adj_{x,t,s}$$

Milliman

Stuart Silverman
*stuart.silverman@milliman.com*

New York
One Pennsylvania Plaza, 38th Floor
New York, NY 10119
USA
+1 646 473 3000 tel

Philip Simpson
*philip.simpson@milliman.com*

London
11 Old Jewry, Third floor
London
EC2R 8DU
UK
+44 0 20 7847 1500 tel

milliman.com